An investment is made with an some expectation of return—growth in value. In reality, in the short term, the value of an investment will fluctuate around some average rate of return. The amount of fluctuation (volatility) in value creates risk—the risk that neither the expected nor the average return will be realized.

An investment can be plotted on a graph of average rate of return versus volatility. The graph to the right shows two investments as circles: one investment in a risk-free asset (e.g. cash or US Treasury bills) and one in an asset with some volatility but higher return (e.g. a stock).
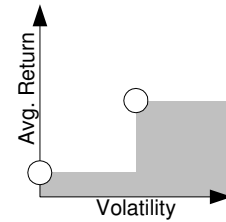
Once the risk and return on investments are known, a rational investor will never choose an investment in the gray areas, because it would have higher risk than known investments, for the same return, or lower return than known investments, for the same risk.

Now consider an investment in a *portfolio,* or mix of the two assets. The volatility and return on the portfolio will be somewhere between the two circles, and closer to the one representing the higher proportion of the investment. In fact, an investment which mixes a volatile asset and a risk-free one will lie somewhere on the straight line between the two circles, as shown in the graph to the right. This extends the gray area, further eliminating investments from rational consideration.
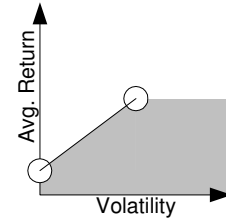

Now, consider an investment in a third asset with higher volatility and return than those considered thus far.

The volatility of a portfolio of two assets depends on the *correlation* of their individual volatilities. If they are not precisely correlated, then downward fluctuations in the value of one asset will be cancelled by upward fluctuations in the value of the other. The portfolio will then lie somewhere on a *curved* line between the two circles, as shown in the graph to the right. Notice that compared to an investment in just the high-risk asset, a small allocation in the lower-risk asset very quickly reduces the volatility of the portfolio. Also, compared to an investment in just the low-risk asset, a small allocation in the higher-risk asset actually *reduces* the volatility while increasing the average return of the portfolio.
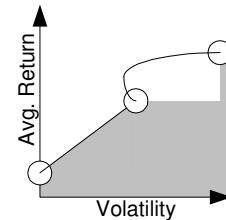
As shown to the right, the curved line further extends the gray area, eliminating more investments from consideration. Note that the straight line now touches a particular point on the curved line. That point represents an investment in a particular mix of two assets, with a particular volatility and return. If reduced volatility is desired, allocation should shift from that mix into risk-free assets, rather than any volatile ones, to maximize average return. Otherwise, the investment will enter the gray area of investments known to be irrational.
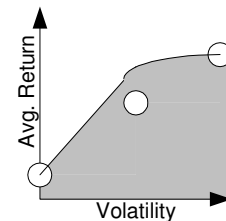

The border between the gray area and the white is called the *efficient frontier.* A rational investor will never choose an investment in the gray area, because it would have higher risk than known investments, for the same return, or lower return than known investments, for the same risk. To invest on the efficient frontier requires the investor to choose an acceptable volatility, and then determine the mix of assets and cash that maximizes average return at that volatility.

The return of a mix of assets is the sum of the returns of each asset, weighted by its proportion in the mix.

The volatility of a mix of multiple assets requires calculation of the correlations of the volatilities of all pairs of assets in the portfolio.

For a given set of assets and a tolerable volatility, once an efficient allocation is known (shown as the black circle in the graph to the right), *borrowed* money (margin) can be invested in the portfolio with an expectation of higher returns—and higher volatility. Such investments lie on the straight line extending the one between the circle for the risk-free asset and the circle for the efficient allocation.
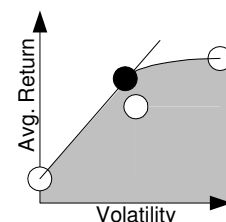
There are two catches: First, for a portfolio consisting of more than two assets, the efficient frontier is a two-dimensional projection of a multidimensional surface, where each dimension represents the allocation of one security in the portfolio. Though the projection may appear smooth, the surface itself may possess "steep mountains and valleys." This means that a small change in the allocation of a security results in a large change in the risk/return of the portfolio. Second, as time passes, the volatility and average return of any component of the portfolio also change. Consequently, it is impossible to calculate and maintain a portfolio that remains on the efficient frontier.